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DMI-ST. EUGENE UNIVERSITY

ZAMBIA

DEGREE EXAMINATION – JUNE 2023

Semester: III 055MA31 ENGINEERING MATHEMATICS I

Time: 3:00 Hours

Max. Marks: 100

Answer any FIVE Questions (5 x 20 = 100 Marks)

- Find the sum and product of the Eigen values of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. (5 Marks)
 - Find the characteristic equation of $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ (5 Marks)
 - If $A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ write A^2 in terms of A and I , using Cayley-Hamilton theorem. (5 Marks)
 - Prove that $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is orthogonal. (5 Marks)
- Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. (10 Marks)
 - Find the value of k if the angle between the lines with direction ratios $-2, 1, -1$ and $1, -k, -1$ is $\frac{2\pi}{3}$. (10 Marks)
- Find the radius of curvature of the curves given by $x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0$ at $(0, 0)$. (5 Marks)
 - Find the envelope of $y = mx + \sqrt{a^2m^2 + b^2}$ where m is a parameter. (5 Marks)
 - Find the evolutes of the parabola $y^2 = 4x$ considering it as the envelope of its normal. (5 Marks)
 - Find the centre of curvature of $y = x^2$ at the origin. (5 Marks)
- Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (10 Marks)
 - Evaluate $\int_0^\infty e^{-ax} \frac{\sin x}{x} dx$ for $a \geq 0$ and hence deduce that (i) $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ and (ii) $\int_0^\infty e^{-x} \frac{\sin x}{x} dx = \frac{\pi}{4}$. (10 Marks)
- Solve the simultaneous equations $\frac{dx}{dt} + 2x + 3y = 2e^{2t}$, $\frac{dy}{dt} + 3x + 2y = 0$. (10 Marks)

b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 3x$. **(10 Marks)**

6. a) Prove that the lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-1}{-2}$ and $\frac{x-2}{2} = \frac{y-3}{-1} = \frac{z+4}{3}$ are coplanar. Find their point of intersection. **(10 Marks)**

b) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 + x - 3y + 2z - 1 = 0$, $2x + 5y - z + 7 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 3x + 5y - 7z - 6 = 0$. **(10 Marks)**

7. a) Solve $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$. **(10 Marks)**

b) Solve $\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$ using method of variation of parameters. **(10 Marks)**