

QP CODE 2020552203

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DMI-ST. EUGENE UNIVERSITY

ZAMBIA

DEGREE EXAMINATION – DECEMBER 2024

Semester: II

055 MA 22 MATHEMATICS II

Time: 3:00 Hours

Max. Marks: 100

Answer any FIVE Questions (5 x 20 = 100 Marks)

1. a) Let $f: \{2,3,4,5\} \rightarrow \{3,4,5,9\}$ and $g: \{3,4,5,9\} \rightarrow \{7,11,15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$. Find $g \circ f$.
(5 Marks)
- b) What are the four steps in formulating a linear programming problem? (5 Marks)
- c) Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_2 \text{ is perpendicular to } L_1\}$. Show that R is symmetric but neither reflexive nor transitive. (5 Marks)
- d) Find $g \circ f$ and $f \circ g$, if $f: \mathbf{sR} \rightarrow \mathbf{R}$ and $g: \mathbf{R} \rightarrow \mathbf{R}$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $g \circ f \neq f \circ g$. (5 Marks)
2. a) If $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then show that (10 Marks)
 $A^3 - 6A^2 + 8A - 3I = 0$.
- b) If the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$, verify that (10 Marks)
 $(A + B)^2 = A^2 + AB + BA + B^2$.
3. a) Check the continuity of the function f given by $f(x) = 2x + 3$ at $x=1$. (5 Marks)
- b) Examine whether the function f given by $f(x) = x^2$ is continuous at $x = 0$. (5 Marks)

- c) Find $\frac{dy}{dx}$, if $x = at, y = 2at$. **(5 Marks)**
- d) Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$. **(5 Marks)**
4. a) Evaluate of the following integral: $\int \frac{dx}{(x+1)(x+2)}$ **(10 Marks)**
- b) Find the following integral: $\int e^x \sin x dx$. **(10 Marks)**
5. a) Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$. **(10 Marks)**
- b) Find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$. **(10 Marks)**
6. a) Let $f' : N \rightarrow R$ be a function defined as $f'(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$, where, S is the range of f , is invertible. Find the inverse of f . **(10 Marks)**

A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating

- b) and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realize a maximum profit? What is the maximum profit per week? **(10 Marks)**

7. a) If the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ **(10 Marks)**
- Find a matrix D such that $CD - AB = O$.

- b) Obtain the inverse of the following matrix using elementary operations $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ **(10 Marks)**