



## DMI-ST. EUGENE UNIVERSITY

## ZAMBIA

## DEGREE EXAMINATION – JUNE 2024

Semester: III

055 MA 32 DISCRETE MATHEMATICS

Time: 3:00 Hours

Max. Marks: 100

Answer any FIVE Questions (5 x 20 = 100 Marks)

1. a) Explain the logical operators. (10 Marks)
  - b) Construct a truth table for the Compound proposition.  $(p \wedge q) \rightarrow (\sim p)$ . (5 Marks)
  - c) Construct the following a truth table for the compound proposition.  $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$  (5 Marks)
2. a) Write the following laws
    - (i) Identity laws
    - (ii) Domination laws
    - (iii) Idempotent laws
    - (iv) Inverse laws
    - (v) Double complement laws (10 Marks)
  - b) Describe the following functions.
    - (i) One-one
    - (ii) onto
    - (iii) Bijection
    - (iv) Partition of a set (10 Marks)
3. a) State and Prove Cancellation laws. (10 Marks)
  - b) Prove that a cyclic group is abelian. (5 Marks)
  - c) Explain the following definitions
    - (i) Group homomorphism
    - (ii) Normal subgroups (5 Marks)
4. a) State and prove The Handshaking theorem. (5 Marks)
  - b) Explain the following types of Subgraphs

(i) Proper subgraph

(ii) Spanning Subgraph

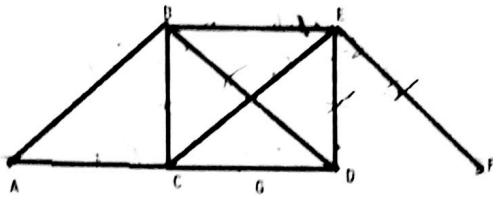
(iii) Vertex deleted subgraph

(iv) Edge deleted subgraph

(v) Induced subgraph.

**(10 Marks)**

- c) Find the number of vertices, the number of edges and the degree of each Vertex in the following undirected graphs. Verify also the hand shaking theorem in each case



**(5 Marks)**

5. a) Prove that the number  $n$  of vertices of a full binary tree is odd and the number of pendant vertices of the tree is equal to  $\frac{n+1}{2}$ . **(10 Marks)**
- b) Prove that an undirected graph is a tree, if and only if, there is a unique simple path between every pair of vertices. **(10 Marks)**

6. a) Prove that

(i)  $A \cap B = B \cap A$

(ii)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**(10 Marks)**

- b) If  $R$  and  $S$  be relations on a set  $A$  represented by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

(i)  $R \cup S$  (ii)  $R \cap S$  (iii)  $R \cdot S$  (iv)  $S \cdot R$

**(10 Marks)**

7. a) If a graph  $G$  (either connected or not ) has exactly two vertices of odd degree ,there is a path joining these two vertices. **(10 Marks)**

- b) Explain the following definitions

(i) Eulerian path

(ii) Eulerian circuit and Eulerian graph

(iii) Hamiltonian path

(iv) Hamiltonian circuit and Hamiltonian graph

**(10 Marks)**