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DMI-ST. EUGENE UNIVERSITY
DEGREE EXAMINATION – DECEMBER – 2022

SEM: VI **055 MA 61 PROBABILITY AND QUEUING THEORY**

Time: 3 Hours Max. Marks: 100

Answer any Five questions (5 x 20 = 100 Marks)

1. a) Find the probability distribution of the total number of heads obtained in four tosses of a balanced coin. Find the Moment Generating Function. **(10 Marks)**

b) Find the moment generating function of the RV X whose probability function

$P(X = x) = \frac{1}{2^x}, x = 1.2 \ldots\ldots$ Hence, find its mean. **(10 Marks)**

2. a) The mean of a binomial distribution is 20 and S.D is 4. Determine the parameters of the distribution. **(5 Marks)**

b) In a binomial distribution consisting of 5 independent trails, probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution. **(5 Marks)**

c) Comment on the following: The mean of a binomial distribution is 3 and variance is 4. **(5 Marks)**

d) The mean and variance of a binomial variate are 8 and 6. Find$p(x \geq 2)$. **(5 Marks)**

3. a) Tests made on the breaking strength of 10 pieces of a metal wire gave the results: 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg. (t test table value v=9, $t_{5\%}$=2.26). **(10 Marks)**

b) A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter 1.85 cm with a SD of 0.1 cm. On the basis of this sample, would you say that work of the machinist is inferior? (t test table value v=9, $t_{5\%}$=2.26). **(10 Marks)**

4. a) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If

the service time for all cars is constant and equal to 10 minutes, determine L_s, L_q, W_s,and W_q. **(10 Marks)**

b) In a given M/M/I queuing system, the average arrivals is 4 customers per minute and $\rho = 0.7$. what are

(i) Mean number of customers L_s in the system,

(ii) Mean number of customers, L_q in the queue,

(iii) Probability that the server is idle, and

(iv) Mean waiting time, W_s, in the system? **(10 Marks)**

5. a) If the transition probability Matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ find the steady-state distribution of the chain. **(10 Marks)**

b) Suppose that the probability of a dry day following a rainy day is $1/3$ and that the probability of a rainy day following a dry day is $1/2$. Given that may 1 is a dry day. Find the probability that May 3 is a dry day and also May 5 is a dry day. **(10 Marks)**

6. a) For the triangular distribution

$f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & otherwise \end{cases}$ Find the moment generating function (MGF) also find cumulative (cdf) distribution function of F(x). **(10 Marks)**

b) Find the moment generating function of the distribution given by $f(x) =$

$\begin{cases} \theta\, e^{-\theta x} & x > 0 \\ 0, & otherwise \end{cases}$

And hence find the moment generating function. **(5 Marks)**

c) Find the moment generating function of the distribution given by $f(x) =$

$\begin{cases} \theta\, e^{-\theta x} & x > 0 \\ 0, & otherwise \end{cases}$ And hence find the M_4. **(5 Marks)**

7. a) Customers arrive at one-man barber shop according to a Poisson process with a mean inter arrival time of 12 min. customers spend an average of 10 min. in the barber's chair.

(i) What is the expected number of customers in the barber shop and in the queue?

(ii) Calculate the % of time of arrival, can walk straight into the barber's chair without having to wait.

(iii) How much time can customer expect to spend in the barber's shop?

(iv) Management will provide another chair and here another barber, when a customer's waiting time in the shop exceeds 1.25h. How much must the average rate of arrivals increase to warrant a second barber? **(10 Marks)**

b) A car mechanic finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs car engines in the order in which they come in ,and if the arrival of engine follows a Poisson distribution approximately with an average rate of 10 per 8-hour day,

(i) What is the mechanic's expected idle time each day?

(ii) How many jobs are ahead of the average engine just brought in?

(iii) What is the average number of jobs in a non-empty queue? **(10 Marks)**