QP CODE	2040554108

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DMI-ST. EUGENE UNIVERSITY
DEGREE EXAMINATION – JANUARY – 2023

SEM: IV **055 MA 41 ENGINEERING MATHEMATICS II**

Time: 3 Hours Max. Marks: 100

Answer any Five questions (5 x 20 = 100 Marks)

1. a) Evaluate: $\int\int xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$. **(10 Marks)**

   b) Evaluate $\int_0^{\log a}\int_0^x\int_0^{x+y} e^{x+y+z}\,dzdydx$. **(10 Marks)**

2. a) Verify Stocks theorem for $\vec{F} = (x^2+y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. **(10 Marks)**

   b) If $\nabla\emptyset = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{j}$; find $\emptyset$ if $\emptyset(-1,2,2) = 4$. **(10 Marks)**

3. a) Show that the function $f(z) = e^z$ is differentiable everywhere in the complex plane. **(5 Marks)**

   b) Prove that an analytic function with constant real part is constant. **(5 Marks)**

   c) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. **(10 Marks)**

4. a) Expand e^z in a Taylor's series about $z = 0$ **(10 Marks)**

   b) Evaluate $\int_c \frac{sin3z}{z+\frac{\pi}{2}}dz$ where c is a circle $|z| = 5$ **(10 Marks)**

5. a) Using gamma integral properties evaluate the following integral: $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{cos^6 x}dx$ **(10 Marks)**

   b) Find the middle term in the expansion of $(\frac{x}{3} + 9y)^9$. **(10 Marks)**

6. a) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate. **(8 Marks)**

   b) If $f(z) = e^z$ then show that u and v are harmonic functions. **(5 Marks)**

   c) Find the image of the circle $|z| = 1$ by the transformation $w = z + 2 + 4i$ **(7 Marks)**

7. a) Solve the following integral $\int_0^2\int_0^{x^2} e^{\frac{y}{x}}dydx$. **(6 Marks)**

   b) Evaluate the following $\int_0^{\frac{\pi}{2}}\int_0^{\infty} \frac{rdrd\theta}{(r^2+a^2)^2}$. **(6 Marks)**

   c) Change the order of integration in $I = \int_0^1\int_{x^2}^{2-x} xy\,dydx$ and hence evaluate it. **(8 Marks)**