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DMI-ST. EUGENE UNIVERSITY

ZAMBIA

DEGREE EXAMINATION – DECEMBER 2023

Semester: IV 055MA42 PROBABILITY AND QUEUING THEORY

Time: 3:00 Hours **Max. Marks: 100**

Answer any FIVE Questions (5 x 20 = 100 Marks)

1. a) A continuous random variable X has p.d.f. $f(x)K, \quad 0 \le x \le 1$. Determine the constant K. Find$P\left[X \le \frac{1}{4}\right]$. **(5 Marks)**

   b) Given that the p.d.f of a R.V X is $f(x) = kx, \;\; 0 < x < 1$ find K and $P(X > 0.5)$. **(5 Marks)**

   c) Suppose that X is a continuous random variable whose probability density function is given by

   $$f(x) = \begin{cases} C \;\; (4x^2 - 2x^2 0 & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

   (i)What is the value of C?

   (ii)Find$P\{X > 1\}$. **(5 Marks)**

   d) A continuous random variable has p.d.f $f(x) = \begin{cases} a + bx, & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$

   If the mean of the distribution is $\frac{1}{2}$ then find a, b. Also find Var(X). **(5 Marks)**

2. a) Find the probability that in tossing a fair coin 5 times, there will appear (i) 3 heads (ii) 3 tails and 2 heads (iii) at least 1 head (iv) not more than 1 tail. **(5 Marks)**

   b) A machine manufacturing screws is known to produce 5% defective, in a random sample of 15 screws, what is the probability that there are (i) exactly 3 defective (ii) not more than 3 defective. **(5 Marks)**

   c) Suppose that$P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3\, Var\,(X)$, find $P(X = 0)$. **(5 Marks)**

   d) If X is a binomial random variable with expected value 6 and variance 2.4. Find $P(X = 5)$. **(5 Marks)**

3. a) The heights of ten males of a given locality are found to be 175, 168, 155, 170, 152, 170, 175, 160, 160 and 165 Cms. Based on this sample, find the 95% confidence limits for the height of males in that locality. **(10 Marks)**

b) Two independent samples of sizes 8 and 7 contained the following values:

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Sample I	19	17	15	21	16	18	16	14
Sample II	15	14	15	19	15	18	16	

Is the difference between the sample means significant? **(10 Marks)**

4. a) The one person barber shop can accommodate a maximum of 5 people at a time (4 waiting an 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5/hr. The barber cuts hair at an average rate 4/hr. (Exponential service time).

(i)What % time is the barber idle?

(ii)What is the fraction of the potential customers are turned away?

(iii)What is the expected number of customers waiting for a hair-cut?

(iv) How much time can a customer expect to spend in the barber shop?. **(10 Marks)**

b) Assume the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 min. If the yard can admit 9 trains at a time. Calculate the probability that the yard is empty and the average queue length. **(10 Marks)**

5. a) Consider the Markov Chain with TPM

$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is it irreducible? If not find the classes. Find the nature of the states. **(10 Marks)**

b) A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain$\{X_n\}$. Find also p^2 and$P[X_2 = 6]$. **(10 Marks)**

6. a) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(i)Find the effective arrival rate at the clinic.

(ii)What is the probability that an arriving patient will not wait?

(iii)What is the expected waiting time until a patient is discharged from the clinic?

(10 Marks)

b) In a single server Queueing system with Poisson input and exponential service times, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum possible number of calling units in the system is 2, then (i) $P_n(n \geq 0)$, average number of calling units in the system and in the queue (ii) Average waiting time in the system and in the queue.

(10 Marks)

7. a) Consider the Markov chain with transition probability matrix given by

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \end{pmatrix}$$

Show that it is ergodic. **(10 Marks)**

b) Consider a Markov chain with state space $\{0, 1\}$ and the transition probability matrix

$$P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

(i) Draw a transition diagram.

(ii)Show that 0 is recurrent.

(iii)Show that 1 is transient.

(iv) Is the state 1 periodic? If so, what is the period?

(v)Is the chain irreducible?

(vi)Is the chain ergodic? Explain. **(10 Marks)**