QP CODE	2040554107

Reg.No: | | | | | | | | | | | |

DMI-ST. EUGENE UNIVERSITY
DEGREE EXAMINATION – DECEMBER – 2022

SEM: IV **055 MA 41 ENGINEERING MATHEMATICS**

Time: 3 Hours Max. Marks: 100

Answer any Five questions (5 x 20 = 100 Marks)

1. a) Calculate the following integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ using polar coordinates. **(10 Marks)**

   b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2y + xy^2) dydx$ **(10 Marks)**

2. a) Verify green's theorem in the xy plane for $\int_c (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(10 Marks)**

   b) Using Gauss-divergence theorem show that $\iint_s \vec{F}.\hat{n}ds = \frac{4\pi}{3}(a + b + c)$. **(10 Marks)**

3. a) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. **(10 Marks)**

   b) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate. **(10 Marks)**

4. a) Calculate the residue of $f(z) = \frac{1-e^{2z}}{z^3}$. **(10 Marks)**

   b) Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where c is $|z| = 3$, using Cauchy's integral formula. **(10 Marks)**

5. a) Using De Moivre's theorem, evaluate$(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6})^{18}$. **(10 Marks)**

   b) Find the term independent of x in the expansion of $(2x + \frac{1}{2x^2})^9$. **(10 Marks)**

6. a) Calculate the residue of $f(z) = \frac{1-e^{2z}}{z^3}$. **(10 Marks)**

   b) Expand e^z in a Taylor's series about $z = 0$ **(10 Marks)**

7. a) Using gamma integral properties evaluate the following integral: $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$ **(10 Marks)**

   b) Find the middle term in the expansion of $(\frac{x}{3} + 9y)^9$. **(10 Marks)**