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DMI-ST. EUGENE UNIVERSITY

ZAMBIA

DEGREE EXAMINATION – JUNE 2023

Semester: VII 760MA07 VECTOR ANALYSIS AND FOURIER ANALYSIS

Time: 3:00 Hours **Max. Marks: 100**

Answer any FIVE Questions (5 x 20 = 100 Marks)

1. a) Given $\mathbf{R} = \sin t\vec{\mathbf{i}} + \cos t\vec{\mathbf{j}} + t\vec{\mathbf{k}}$, find $(a)\,\frac{d\mathbf{R}}{dt}$, (b) $\frac{d^2\mathbf{R}}{dt^2}$, (c) $\left|\frac{d\mathbf{R}}{dt}\right|$, (d) $\left|\frac{d^2\mathbf{R}}{dt^2}\right|$. **(5 Marks)**

   b) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$. **(10 Marks)**

   c) For the curve $x = e^t\, cost\,, y = \sin t,\ z = e^t$. Find the velocity and acceleration of the particle moving on the curve at $= 0$. **(5 Marks)**

2. a) If $\phi(x, y, z) = x^2y + y^2x + z^2$ find $\nabla\phi$ at the point (1,1,1) **(5 Marks)**

   b) If $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ and $r = |\vec{r}|$ prove that $\nabla \mathrm{r} = \frac{1}{r}\vec{r}$ and $\nabla\frac{1}{r} = \frac{-\vec{r}}{r^3}$. **(10 Marks)**

   c) If $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ and $r = |\vec{r}|$ prove that $\nabla(\log r) = \frac{\vec{r}}{r^3}$ **(5 Marks)**

3. a) If $\vec{f}(t) = (3t^2 - 1)\vec{\imath} + (2 - 6t)\vec{\jmath} - 4t\,\vec{k}$ find $\int_2^3 \vec{f}(t)dt$. **(5 Marks)**

   b) The acceleration $\vec{a}$ of a particle at any time is given by $\vec{a} = e^{-t}\vec{\imath} - 6(t + 1)\vec{\jmath} + 3\sin t\,\vec{k}$.

   If the velocity and displacement are zero at time $t=0$ find the velocity and displacement vector. **(5 Marks)**

   c) Prove that $\int \vec{r} \times \frac{d^2\vec{r}}{dr^2}\, dt = \vec{r} \times \frac{d\vec{r}}{dt} + \vec{c}$ and evaluate $\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2}\, dt$ where $\vec{r} = 5t^2\vec{\imath} + t\vec{\jmath} - t^3\vec{k}$.

   (10 Marks)

4. a) Obtain a Fourier series expression for e^x in the interval $-\pi < x < \pi$. **(5 Marks)**

   b) Obtain a Fourier series for $f(x) = \begin{matrix} \frac{1}{2} & -\pi < x < 0 \\ & 0 < x < \pi \end{matrix}$ Deduce $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots\dots\dots. = \frac{\pi}{4}$.

   (10 Marks)

   c) Show that foe all values of x on $(-\pi, \pi)$, $\frac{x}{2} = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots \quad \infty$.

   (5 Marks)

5. a) Prove that $F[\overline{f(x)}] = \overline{F(-s)}$. **(5 Marks)**

b) Show that $F[\int_a^x f(x)dx] = \frac{F(s)}{(-is)}$. **(5 Marks)**

c) Find the Fourier transform of $f(x) = \begin{cases} \cos x & if\ \ 0 < x < 1 \\ 0 & otherwise \end{cases}$. **(10 Marks)**

6. a) Find the acceleration of the particle which moves along the curve $x = 2\sin 3t, y = 2\cos 3t, z = 3t\ at\ \ t = \frac{\pi}{2}$. **(5 Marks)**

b) If $\vec{r} = \vec{a}\cos\ \omega t + \vec{b}\sin\omega\ t\quad show\ that\ \ \vec{r} \times \frac{d\vec{r}}{dt} - \omega\ \vec{a} \times \vec{b},\ \frac{d^2\vec{r}}{dt} = -\omega^2\ \vec{r}$ **(10 Marks)**

c) If $\mathbf{A} = 5t^2\boldsymbol{i} + t\mathbf{j} - t^3\vec{\mathbf{k}}$ and $\mathbf{B} = \sin t\vec{\mathbf{i}} - \cos t\vec{\mathbf{j}}$, find $\frac{d}{dt}(\mathbf{A} \cdot \mathbf{B})$. **(5 Marks)**

7. a) If $\vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$. Evaluate $\int \vec{F}.d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from $(0,0)\ to\ (1,2)$. **(5 Marks)**

b) If $\vec{F} = (3x^2 + 6y)\vec{\imath} - 14yz\vec{\jmath} + 20xz^2\vec{k}$ evaluate $\int \vec{F}.d\vec{r}$ from $(0,0,0)\ to\ (1,0,0)$ along the following paths.

(i) $x = t,\quad y = t^2,\quad z = t^3$

(ii) The straight lines from $(0,0,0\ to\ (1,1,0)$ then to $(1,1,0)\ \ and\ then\ to (1,1,1)$.

(iii) The straight line joining $(0,0,0)$ and $(1,1,1)$. **(10 Marks)**

c) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\vec{\imath} - 5z\vec{\jmath} + 10x\vec{k}$ along the curve $x = t^2 + 1,\ \ y = 2t^2,\ \ z = t^3$ from $t = 1\quad to\quad t = 2$. **(5 Marks)**