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DMI-ST. EUGENE UNIVERSITY

ZAMBIA

DEGREE EXAMINATION – DECEMBER 2023

Semester: II **055MA22 MATHEMATICS II**

Time: 3:00 Hours **Max. Marks: 100**

Answer any FIVE Questions (5 x 20 = 100 Marks)

1. a) A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit? (**10 Marks)**

b) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A? **(10 Marks)**

2. a) If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ Calculate AC, BC and (A+B)C. Also, verify that (A+B)C=AC+BC **(10 Marks)**

b) Find X and$Y$, if $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$. **(10 Marks)**

3. a) Evaluate the left hand and right hand limits of the function $f(x) = \begin{cases} \frac{|x-3|}{x-3} & if\ x \neq 3 \\ 0 & if\ x = 3 \end{cases}$ at $x = 3$. What do you conclude? **(10 Marks)**

b) Show that $f(x) = \begin{cases} 5x - 4 & if\ \ 0 < x \leq 1 \\ 4x^3 - 3x & if\ \ 1 < x < 2 \end{cases}$ is continuous at $x = 1$. **(10 Marks)**

4. a) Evaluate $\int \frac{x+3}{(x+2)^2(x+1)} dx$. **(10 Marks)**

b) Evaluate the following integrals:

(i) $\int 2x\sqrt{1+x^2}dx$ (ii) $\int x(a-x)^8 dx$ **(10 Marks)**

5. a) A train started from Madurai Junction towards Coimbatore at 3pm (time $t = 0$) with velocity $v(t) = 20t + 50$ kilometer per hour, where t is measured in hours. Find the distance covered by the train at 5pm. **(10 Marks)**

b) A tree is growing so that, after t - years its height is increasing at a rate of $18/\sqrt{t}$ cm per year. Assume that when *t* = 0, the height is 5 cm.

(i) Find the height of the tree after 4 years.

(ii) After how many years will the height be 149 cm? **(10 Marks)**

6. a) Find the intervals in which the function *f* given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(i) Strictly increasing

(ii) Strictly decreasing **(10 Marks)**

b) Find the area of the region bounded by the two parabolas $y = x^2$ and$y^2 = x$.

(10 Marks)

7. a) Find the second order derivative of the following functions with respect to x.

(i) $x = at^2, y = 2at$

(ii) $x \sin x$ **(10 Marks)**

b) Using the Rolle's theorem, determine the values of *x* at which the tangent is parallel to the *x* -axis for the following functions:

(i) $f(x) = x^2 - x,\ x \in [0,1]$

(ii) $f(x) = \frac{(x^2-2x)}{(x+2)},\ x \in [-1,6]$ **(10 Marks)**